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Differentiable and Piecewise Differentiable Gaits for Snake Robots

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Abstract—**This paper describes a series of gaits which we developed for a free crawling snake robot. Snake robots, a class of hyper-redundant mechanisms, can use their many degrees of freedom to achieve a variety of locomotion capabilities. Like their biological counterparts, snake robots locomote using cyclic motions called gaits. These cyclic motions directly control the snake robot's internal degrees of freedom which causes a net motion (e.g. sining moves the robot forward, strafing moves the robot laterally, and spinning rotates the robot about its center). The gaits described in this paper fall into two categories: differentiable and piecewise differentiable. The differentiable gaits, as their name suggests, can be described by a differentiable function whereas the piecewise cannot. This paper describes the functions we prescribed for gait generation and our experiences in making these robots operate in real experiments.**

I. Introduction

In recent years, a class of robots called snake robots have gained attention. Their potential to locomote in a variety of terrains suggests a number of practical applications. These robots, also called *hyper-redundant mechanisms* [1], [2], inherit their maneuverability from their unique shape changing capabilities. Their long and slender shape allows them to thread through tightly packed volumes without disturbing surrounding areas. This shape also allows actuators to be distributed down the length of the device. Such a distribution enables a greater range of interaction between the environment and the mechanism, as compared to conventional wheeled devices and recently developed legged machines.

There have been a variety of hyper-redundant robots built in recent years, which can be divided into two classes. The first class are those that are mounted on either a fixed base [3], [4], or a mobile base [5], [6] to increase their reach. The remaining robots, which we hereby refer to as snake robots, locomote without the aid of a fixed base. These types of robots either rely on a moving tread or skin [7], [8] or internal shape changes [9]-[11].

We are interested in designing and programming snake robots that rely solely on internal shape changes to locomote. A number of works have been written [12], [13], which describe how the internal shape configuration variables cause motion in the position configuration variables for a broad class of mechanical systems. These works focus on systems that are complicated enough to offer a richer understanding of basic science but simple enough that they are tractable. Unfortunately, these efforts are not quite ready for high degree of freedom systems (i.e. snake robots).

Mark Yim, a researcher at the University of Pennsylvania, has done work on modularity. Yim constructed a modular snake robot made up of single degree of freedom modules called polypods. Yim's contribution is not the snake robot itself but the modularity of his mechanism. He has shown his modular robots work successfully in a variety of terrains [14]. The distribution of actuators in snake robots lends itself well to modularity.

Our group, led by Professor Howie Choset at Carnegie Mellon University, developed a modular snake robot based on Yim's design ideas. Each module contains a standard hobby servo with electronics replaced to give the servos more power and to enable addressability. The group's real contribution, however, is the vast array of behaviors that endow these robots with practical capabilities, including but not limited to crawling on the ground, strafing laterally, swimming, and climbing up the inside and outside of pipes and poles. The pole climbing work has recently received considerable attention. See [figure 1].

Figure 1: Our locomoting robots: reaching across a gap, traversing through brush, the inside of a pipe, and swimming.

II. Behavior Architecture

When biological snakes lost their limbs they acquired unique locomotive capabilities that outperform "conventional" legged animals in cluttered three-dimensional spaces. Likewise, hyper-redundant robots can have comparable locomotive capabilities that outperform wheeled and legged robots in difficult-to-negotiate three-dimensional terrains.

The challenge in designing effective snake robot gaits is coordinating the internal degrees of freedom such that the snake robot moves in a controlled manner. For a 16 DOF robot, this coordination requires planning in a 16-dimensional non-Euclidean configuration space. We address this challenge following Chirikjian and Burdick's [2] backbone approach, where the mechanism is fit to a one-dimensional curve embedded in a three-dimensional space. At first glance, it may seem that a real snake's movement follows a sinusoid, i.e. their backbone curves are sinusoids. Hirose showed that biological snakes actually follow a serpenoid curve for lateral undulation, which is different from a sinusoid but still periodic [15].

Sinuosoids, the serpenoid, and all of our basic gaits fall into a class of gaits called differentiable gaits – named as such because they can be described by a differentiable function. Instead of specifying a differentiable function to which our snake robot should be fit, we sample many configurations along a differentiable curve. By sequencing through these configurations, the robot's joint motions approximate a differentiable curve which has the effect of propelling it forward. Our use of hobby servos necessitates our choice of sequencing through configurations, for these servos only allow us to specify position and not velocities.

Many times a task may require a more complicated set of motions than a single differentiable gait can provide. For example, our current version of stairclimbing requires the robot to repeat a sequence of distinct motions: move along the step, reach up and over the step, and finally coil onto the higher step. In this case three distinct differentiable motions are patched together form a stairclimbing gait. We describe these types of complex gaits as piecewise differentiable because the time varying backbone curve they approximate is piecewise differentiable.

The issue then becomes how to specify all of the joint configurations, which the gait designer has to do, while checking that the actuators can servo to the desired positions. The control architecture for the robots was specifically designed to facilitate the completion of this task.

Our robot's mechanical architecture consists of a series of modules, usually sixteen, where each module contains a revolute joint with a range of motion of 180 degrees. To produce motion in two orthogonal planes, the axes of rotation of any two adjacent modules are offset by 90 degrees. The modules' electronics allow us to send commands to each individual module and to drive the motors using PID control with trapezoidal velocity profiles. The snake operator controls the snake with a keyboard or joystick through a computer interface. Keyboard and joystick commands are used to initiate and modify the execution of a gait. The interface gives a real-time feedback about each servo to the snake operator, including the desired and actual joint angles, the current draw, and the internal temperature.

III. DIFFERENTIABLE GAITS

Differentiable gaits are useful since they can be described by a small set of parameters, thus allowing for a concise representation. All of our differentiable gaits are based upon sinusoidal motion in two axes.

For the purpose of discussing the implementation of these gaits, we assume that the robot's modules are numbered in increasing numerical order (0 starting) from front to back. We also assume that the robot starts with the evenly numbered modules' joint axes parallel to the ground, such that if these even modules are bent, sections of the snake will be lifted off of the surface. If a vertical wave perpendicular to the ground is sent through the snake, only the even modules participate, while the odd modules' joint angles do not change. In contrast, the remaining, odd numbered modules contain joint axes perpendicular to the ground. Only the odd modules participate in sending lateral waves parallel to the ground through the robot. Thus, we refer to these sets of even and odd modules as the 'vertical' and 'lateral' modules, respectively, based upon the modules' participation in either vertical or lateral waves in the robot's body shape. Should the robot fall onto its side, the even modules simply become the lateral modules and the odd become the vertical modules. Given this assumption that the even modules are responsible for propagating vertical waves and the odd for propagating lateral waves, we use the terms $offset_{vertical}$, $offset_{lateral}$, $amplitude_{vertical}$, and $amplitude_{lateral}$ to denote the parameters affecting the two orthogonal waves. Finally, we assume that both offset parameters are set to zero, unless otherwise noted.

Although theoretically continuous, for implementation purposes we sample time at discrete intervals while executing a gait. Where n is the module number and t is time, all of our differentiable gaits can be described by

$$angle(n,t) = \begin{cases} offset_{even} + amplitude_{even} * \sin(\theta), n = even, \\ offset_{odd} + amplitude_{odd} * \sin(\theta + \delta), n = odd, \end{cases} \quad (1)$$

$$\theta = \left(\frac{d\theta}{dn} * n + \frac{d\theta}{dt} * t \right), \quad (2)$$

where offset is the central angle value of the wave, typically zero degrees. A boundary check is later performed on all angle values to ensure that they fall within +/- 90 degrees, the mechanical limitations of the joints.

The offset parameter is generally used to aim the robot when locomoting on the ground. For instance, when the robot is straight or climbing a pipe, the offset is zero. The amplitude terms describe the amplitudes of the mutually perpendicular waves. In most cases, the amplitude and speed of the robot are directly correlated and as both increase, the robot can better reach over large obstacles. However, larger amplitude comes at the cost of reduced stability. Intuitively, given a fixed number of modules, the fewer waves in the robot, the larger each wave can be. Thus, smaller values of $d\theta / dn$ allow more modules to participate in any one wave, which in turn allows for larger waves.

The frequency of the sine wave is determined by θ, which increases with time and along the length of the snake robot. The rate at which θ changes is controlled by the $d\theta / dn$ and $d\theta / dt$ terms. Note that the sine wave moving through the robot has both a spatial and temporal component, determined by $d\theta / dn$ and $d\theta / dt$ respectively. Note that we can determine the angle of every module at any given instant by freezing time (i.e. fixing t) and evaluating the function for each module n. Similarly, we can plot the movement of an individual actuator over time by fixing the actuator number n and evaluating the angle function at desired time values t. Setting $d\theta / dn$ to zero causes identical angles on all of the modules on one axis, generating a constant radius arc along that axis, while setting $d\theta / dt$ to zero has the effect of freezing time.

Finally, δ is simply a phase offset to control the timing between the motions of the two orthogonal waves. These parameters can be modified to change the type of and nature of the gait performed by the robot.

A. Biologically Inspired Gaits

Real snakes exhibit many forms of locomotion. We were inspired by the biological gaits of linear progression, sidewinding, concertina, and lateral undulation when designing our robotic gaits. Our snake robots lend themselves nicely to linear progression, and hence it is our most widely used gait. Linear progression is motion parallel to the length of the snake performed with forward moving waves [16]. Inchworms exhibit this motion with one very large wave, whereas biological snakes use numerous miniscule waves. In our model, the amplitude of the lateral wave is set to zero so that only the modules of the vertical axis execute a sine wave. Modules are picked up from the rear of the robot, brought forward through the air, and placed upon the ground again as the waves progress [Figure 2]. Though larger amplitudes and longer waves (as controlled by $d\theta / dn$) produce faster locomotion and increase the ability of the snake to clear obstacles, we have found that it is generally preferable to use many small waves for stability purposes.

While executing linear progression, the waves cause the robot's center of gravity to rise. This, combined with the fact that the robot's footprint is simply a straight line, causes the robot to occasionally tip over sideways. To stabilize the robot, we position the lateral modules to widen the robot's footprint. The simplest way to perform this correction is to set $\text{offset}_{\text{lateral}}$ to a non-zero value, causing the lateral modules to form a stable arc while the vertical modules continue to propel the robot with waves. This arc shape has the additional benefit of allowing the operator to steer the robot by changing the radius and direction of the arc. An alternative stable footprint which still allows perfectly straight motion is an 'S' shape, which is generated in the lateral axis with two opposite arcs in the front and back half of the snake.

To achieve linear progression around corners in situations where there is not enough room to use a smooth arc, such as inside pipes, we create a sharp bend in the robot on the lateral axis that conforms to the corner. To continue conforming to the corner while the robot moves forward, the bend is moved backwards relative to the robot. The bend must be transitioned gradually from the n to the n+2 (next lateral) module, synchronized with the progress of the robot such that the total bend angle remains contoured to the corner.

Linear progression also enables the robots to climb vertical channels. The amplitude of the wave must be large enough that the robot pushes outwards against the walls. This climbing gait surpasses concertina locomotions for two reasons: 1) momentum is conserved as waves move forward through the robot and 2) concertina locomotions tend to focus stress over only a few modules. If the channel that the robot is climbing is sufficiently deep (i.e. parallel walls), the robot can be steered in and out of the channel using a lateral arc, the same way it can be steered left and right on the ground.

Linear progression can also be used to climb the inside of pipes, similar to how the robot climbs channels. However, since pipes, unlike channels, are fully enclosed, linear progression can be enhanced specifically for pipe climbing. With δ set to zero and with equal amplitudes for both axes, the physical shape of the robot will model a sine wave along a diagonal between the two axes. This "double linear progression" allows all 16 modules to participate in the wave, as opposed to only the 8 "vertical" modules typically used to propel the robot. With double linear progression, the robot is able to climb wider pipes than is possible with single linear progression, and is also able to climb more quickly.

Sidewinding is another biological gait that has inspired our robot gaits. Identical vertical and lateral waves, with $\delta = \pi / 4$, are used to perform sidewinding, which we refer to as "strafing" due to the sideways nature of the motion. The specially chosen value of $\delta = \pi / 4$ causes the robot to move almost directly sideways because the lateral wave begins to translate modules to the side just as they are at the peak of the vertical wave, where the modules are free from friction with the ground [figure 2]. The vertical wave then sets the modules back down just before the lateral wave returns to the initial position. However, now that they are on the ground, the lateral modules are restricted by friction. The fact that the modules experience friction when moved in one direction but not the other leads to a net force from the ground to the robot in the desired direction. In this manner, the snake locomotes sideways, orthogonal to its length. By modifying the value of δ , the angle at which the snake moves sideways can be controlled. This is useful to achieve motion along a diagonal. As with linear progression, the amplitude controls how aggressively the gait is performed, while $d\theta / dn$ controls the number of waves in the robot. Strafing's inherent stability due to its wide footprint allows the gait to be run with large amplitudes, allowing rapid progression and traversal over obstacles. Having the front and back halves of the snake strafe in opposite directions produces an alternative use for the strafing gait. The resulting motion will be the robot rotating in place about its center.

The most common form of locomotion in biological snakes is lateral undulation. A gait called slithering is produced by having the lateral axis execute a sine wave with half of the frequency of the vertical wave. This gait can be conceptualized as simple linear progression, except the lateral wave is continually undulating the snake. Slithering looks a lot like lateral undulation in that the lateral wave is larger and more prominent than the vertical wave. In reality, slithering is an approximation of lateral undulation which also has components of linear progression.

Snakes may have inherited their lateral undulation gait from fish that use lateral undulation to swim. In fact, biological snakes exhibit lateral undulation as a means of swimming [17]. Our snake robots can also swim using lateral undulation in water. In order to swim, we place the snakes in a waterproof skin and run them wirelessly with battery power. To swim, we perform true lateral undulation in that all of the propulsion comes from sinusoidal motion in the lateral axis.

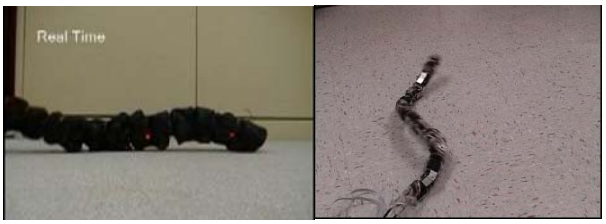

Figure 2: Linear progression and strafing differentiable gaits demonstrated by two different snake robots.

B. Non-biologically Inspired Gaits

We also derived a number of gaits not inspired by biological snakes. Our inspiration for developing these gaits was to achieve specific tasks quickly and reliably. One such gait is rolling, where the robot forms a smooth arc and then rolls side over side across the ground. To achieve this motion, $d\theta / dn$ is set to zero such that at any given instant in time, each axis conforms to an arc. δ is set to $\pi / 2$, creating a full wave offset between the waves of the two axes. When the angles of one axis are at their maximum ($\theta = k\pi$), the angles of the other axis will be zero ($\theta = k\pi + \pi / 2$). These two arcs, one at a maximum while the other is at zero, combine to make an arc incorporating all modules for any value of t. Amplitude in this case controls the radius of curvature of the arc. As time increases, the arc rolls across the ground.

With a non-zero value of $d\theta / dn$, the snake will model the shape of a helix. This helical configuration will roll while maintaining the helix shape, as can be seen in Figure 3. This motion is used to climb up the outside of poles and the inside of pipes. The width of the helix and the helical pitch angle can be modified with amplitude and $d\theta / dn$. Since this pole-climbing gait is only a small variation of the rolling gait, the robot is capable of transitioning between rolling on the ground and climbing up a pole. To the best of our knowledge, we are the first and only robot that can climb the outside of poles using undulatory motions.

Figure 3: The snake robot forms a helix and climbs the outside of a pole.

IV. Piecewise Differentiable Gaits

Many tasks are difficult to complete solely by executing a differentiable gait. These cases arise due to task complexity and robot hardware limitations such as size and motor strength. In these cases a piecewise differentiable gait is used. Unlike differentiable gaits, which are designed to locomote in a wide range of terrains, piecewise differentiable gaits are developed specifically for the robot to complete a desired task. Examples of such tasks include stairclimbing, gap crossing, reaching into a hole in a wall, railroad track crossing, and camera scanning with the head of the robot.

One of the challenges we must overcome in the design of piecewise differentiable gaits has to do with limitations in motor strength. For example, in stairclimbing, the actuators are not strong enough to simply lift a major portion of the robot into the air and place it onto the next step. Instead, we lift a few modules at a time to keep a small perpendicular moment arm, so that we can eventually reach onto the top of a stair.

To lift many of the robot's modules into the air, the robot at all times must balance and can not tip in any direction. To accomplish this, we use the remaining modules on the ground to form a wide, stable base. Once enough of the modules are on the top of the step, the modules at the bottom of the stair work together with those at the top to push and pull more of the robot mass to the top of the stair. Those modules at the top of the step must be carefully positioned to avoid tipping away from the staircase. This entire process may look like rolling or corkscrewing up the step. The final portions of the motion take the remaining modules on the lower stair and lift them into the air using the stable base now in position at the top of the step. After the entire robot is onto the surface of the higher step, a differentiable gait is used to position the robot in preparation for the next step. Using this process, an entire flight of stairs can be scaled.

Since the configurations to achieve this stairclimbing are generated by a gait designer with a specific sized stair in mind, the process is brittle to changes in step size.

Additionally, the piecewise differentiable gait may fail to successfully climb a stair if the robot is not properly aligned to the stair when the gait is initiated, either due to being angled or being too far away from the edge of the stair. See Figure 4.

Figure 4: A snake robot using a piecewise differentiable stair climbing gait.

A second instance of a piecewise differentiable gait that we successfully developed is gap crossing. Linear progression is used to drive the robot over the edge of the gap. The modules that are overhanging the gap are bent downwards into the gap, whose weight assist in pulling the robot forward towards the gap. Once enough modules are down in the gap, we bend them to the side before lifting them out of the gap and into the air. Bending the modules to the side allows the rest of the snake to lift the bent modules with less torque, and prevents them from colliding with the far side of the gap. Once the front modules are lifted, the driver straightens the bend, which extends the lifted modules over the gap. When lowered, these extended modules make contact with the ground on the other side of the gap, resulting in the robot having successfully bridged the gap.

From this position, the driver again uses linear progression to move the robot forward across the gap. So long as the robot has reasonable contact on both sides of the gap, this gait effectively moves the robot forward. As less of the robot is in contact with the starting side of the gap, linear progression becomes ineffective. At this point, the driver bends the lateral modules on the far side of the gap to form a stable base, which allows the driver to lift the very back modules of the robot into the air and pull them over the gap using the midsection of the snake.

When used with a 36” sixteen module snake, this gap crossing piecewise differentiable gait is effective for any gap ranging 6” to 13”. This flexibility is in part due to the operator control of the gait during the linear progression portions of the gap crossing process. Gaps shorter than 6” can be easily crossed with a simple differentiable gait such as linear progression.

As discussed, tasks such as stairclimbing and gap crossing require complicated processes to complete. A gait designer must identify a broad “strategy” for completing the task and then design the series of configurations to reach these goals, whilst always considering motor strength and stability issues.

V. Conclusions and Future Work

This paper describes a series of gaits for a snake robot comprising single degree of freedom modules daisy chained together where each axis is rotated orthogonally to the previous. Inspired by biology and based on our empirical experiences with these devices, we have been designing gaits for these robots for several years and have had some impressive demonstrations. In this paper, we describe the insight behind developing these gaits. A companion paper describes the architecture for this robot [18].

In addition to continuing work with the gaits, described in this paper, our lab is working on improving the mathematical model on which the gaits are based in an attempt to generate novel periodic gaits. One way we would like to improve the gait generation model is by introducing the ability to add sinusoidal waves together. The ability to sum sine waves permits us to generate such periodic functions as square waves. We are attempting to use this method to discover the most effective periodic functions to use as differentiable gaits.

A second way we aim to improve our differentiable gaits is by varying the wave parameters. Although the model described in this paper holds the wave parameters (i.e., amplitude, offset, etc.) constant, results from early experimentation show that varying some of these parameters as a function of time or module number proves useful. For instance, we have observed that the slithering gait is more effective when we linearly increase the lateral wave amplitude from the head to the tail of the robot. Soon we hope to apply these results to develop gaits that allow the robot to climb tapered channels, pipes, and poles. We also plan to apply this development to situations such as transitions between gaits

Other future gait development work involves fitting sinusoidal curves to desired position data. Unlike our existing differentiable gaits, where the robot operator specifies wave parameters resulting in sinusoidal positions, piecewise differentiable gaits are designed by specifying positions for each actuator. Fitting sinusoidal curves to these specified positions will enable us to easily modify piecewise differentiable gaits. For example, currently if we need to adapt a piecewise differentiable stair-climbing gait for a different stair height, a gait designer needs to change all of the actuator positions associated with the gait, to a large extent writing an entire new gait. This new method would allow for the entry of parameters appropriate to the behavior, such as stair height, to control the amplitude of the approximation function to easily and instantly modify the gait.

Another avenue of future research to enable flexibility is using sensor feedback from the robot to ensure that the robot fulfills higher level goals in uncertain environments. For instance, the gait developer may specify that during stairclimbing it is essential for a certain section of modules to remain vertical. Sensor feedback could allow the robot to adjust itself to prevent tipping. Through effective use of sensor feedback (touch sensors, accelerometers, current sensing), piecewise differentiable gaits can use higher level goals adaptable to many situations, as opposed to being a hard coded list of ordered configurations.

This sensor feedback may prove useful for differentiable gaits, as well. Currently, when climbing the outside of a pole, the operator tightens the helix radius until the robot gains sufficient traction. Through the use of current and position sensing, the robot itself could determine proper helix pitch and radius to optimally climb a pole with respect to speed or power efficiency. Furthermore, sensors can be used to provide the robot with semi-autonomous capabilities, such as maintaining balance and orientation as the operator directs the robot with a differentiable gait.

We are constantly researching new gaits, both differentiable and piecewise differentiable. Our future work will not only improve our gaits, but also improve the gait generation process. We will continue to work towards robust and reliable locomotion in a broad range of environments and to further utilize the flexibility of our hyper-redundant robots.

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