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COORDINATES MATTER: VIRTUAL CHASSIS AND MINIMUM PERTURBATION COORDINATE REPRESENTATIONS

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Although any set of generalized coordinates can be used to represent the motion of a system, the choice of coordinates can greatly effect the ease of its analysis. Notably, the choice of body frame can affect a system's representational clarity as well as the computational accuracy of any linear approximations. In this paper we examine how departing from the typical convention of rigidly fixing a body frame to a system and instead carefully choosing the body frame can provide these benefits to a number of articulated locomoting systems, namely modular snake robots and planar swimming systems.

1. Introduction

In principle, any set of generalized coordinates can represent the motion of a system. In practice, the choice of coordinates can greatly effect the tractability and the ease of analysis of that system. For example, if the body frame of a car were fixed to one of its wheels, instead of its chassis, the mathematical expressions of the car's motion through the world would be significantly more complex and more difficult to understand. Similarly, if we consider a satellite floating in space, assigning a body frame to the center of mass simplifies the analysis because, by conservation of momentum, the origin cannot translate as a result of internal shape changes.

Recently we have explored this topic in two contexts. The first is with theoretically-driven planar locomoting systems using a framework that generalizes the center-of-mass analysis of inertially-isolated systems into a broader notion of *minimum-perturbation* coordinates.[1] The second is with real-world systems, such as snake robots,[2–5] that do not have a stable point on the body analogous to a wheeled vehicle's chassis. In this case, we calculate an averaged body frame to serve as a *virtual chassis*.[6]

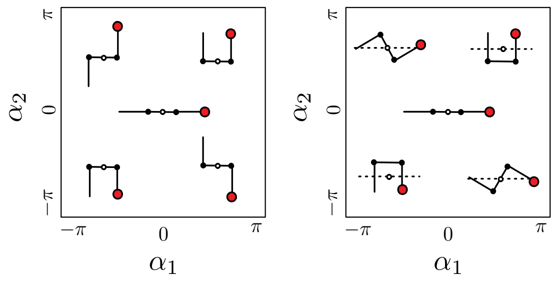

Fig. 1. Diagrams of the three-link swimming snake shown at zero orientation across corresponding shapes in a body frame fixed to the middle link (left) and a body frame defined by mean orientation (right).

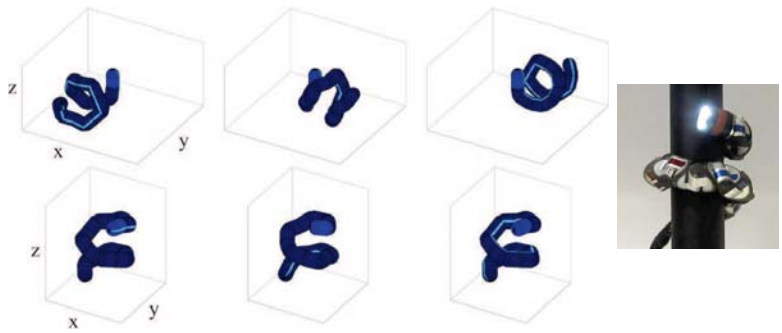

Fig. 2. A montage showing a snake robot at 3 positions in a gait used for climbing poles (right) viewed in a frame fixed to the head module (top row) and a frame aligned with the virtual chassis (bottom row).

In this paper we unify the results from these two contexts, show that same underlying philosophies are at work, and show that our theoretically grounded results with minimum perturbation coordinates can help us to better understand the benefits of using the virtual chassis on a real system. Our philosophy is built on two subsequent ideas. The first is that the motion of a locomoting system should be studied primarily from the perspective of that system's body frame, as opposed to an external world frame. The second is that the choice of body frame has a significant effect on the mathematics and techniques that can be brought to bear in studying that system.

2. Coordinate Choice

The roots of our investigation of coordinate choice lie in the observation that the overall motion of an articulated system is often simpler and more intuitive than the motion any single part of that system. Based on this observation, we propose that a "good" body coordinate frame is one that represents a system's overall external motion independently from its internal shape changes. For articulated locomoting systems, this means departing from the typical convention of rigidly fixing a body frame to the system and instead defining it with respect to the system's constraints or overall shape.

A key result from our prior geometric work is that good choices of body frame are those for which the pose of the system fluctuates the least in response to internal changes in shape. For planar locomoting systems, this can be formalized by optimizing our frame such that differential changes in position induced by differential changes in shape as small as possible at all the points in our shape space. Specifically, this is accomplished using the *Hodge-Helmholtz Decomposition*,[7] which separates the vector fields that represent the system's motion into a conservative component and a divergence-free remainder. We term such frames *minimum perturbation* body frames.

For real-world systems, calculating the minimum perturbation coordinates is rarely feasible, since we lack a model for how the robot moves in response to its shape changes. However, experience and intuition have shown that the minimum perturbation coordinates are often close to those defined by the center of mass and mean orientation of the system. Therefore, for our lab's snake robots, we calculate the *virtual chassis* by continually aligning the body frame with the robot's principle moments of inertia. Specifically, this is done by taking the singular value decomposition (SVD) of the positions of all of the robot's links with respect to the robot's center of mass at discrete time steps as the robot executes the gait.

Unlike the minimum perturbation coordinates, the virtual chassis does not explicitly take into account the robot's interaction with the world. However, in practice it is a good approximation to how the robot's orientation changes in response to internal shape changes and we see benefits similar to those provided by the minimum perturbation coordinates. These benefits can be characterized as providing either better representational clarity or improved computational accuracy.

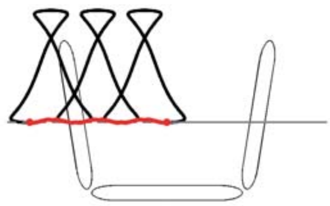

Fig. 3. The path traced by a three-link swimmer, represented in a body frame fixed to the middle link (black line) and a minimum perturbation body frame (red line). Note that the black path has been shifted vertically so that the start and stop locations are aligned.

3. Representational Clarity

The virtual chassis and minimum perturbation frameworks provide representational benefits that allow us to describe the motion of articulated systems in an intuitive and natural way.

An example of this improved representation involves the three-link planar swimming snake. Figure 1 shows a swimmer at zero orientation 5 different internal shape configurations. Unlike the middle-link body frame, the minimum perturbation representation takes into account the constraints of the swimmer, and as such it has the property that moving from the center shape to any of the 4 outer shapes would leave the system in a configuration whose pose is almost the same. Figure 1 shows the configuration of our lab's modular snake robot in 3 frames of the pole climbing gait. The robot's motion in the virtual chassis frame stays oriented with the pole, rather than the head when using a fixed frame. These effects come from some key traits of this representation:

(1) As an average point on the system, the body frame moves in a straight line with approximately constant velocity as the system executes gaits.
(2) Similarly, the body frame rotates very little, meaning that as the robot executes a gait, the direction it translates per cycle is independent of the starting phase. Together with the previous benefit, this means that executing a partial cycle of a gait moves the system a proportional fraction of the motion over a full gait cycle.
(3) The swept volume of the robot's body over the range of shapes corresponding to a given pose is minimized.

The first and second traits are highlighted by the example of the three-link swimmer in Fig. 3, which shows the paths of the exact same motion

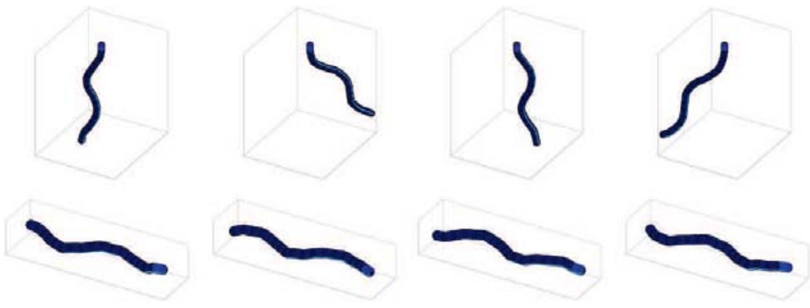

Fig. 4. A montage of the robot in sidewinding, shown in different positions from one gait cycle. The top row of images show the pose of the robot in a body frame fixed to the head link. The bottom row shows the pose corresponding to the virtual chassis body fame at the same points in the gait cycle.

viewed in different body frames. The pose represented by the minimum perturbation body frame changes monotonically towards its final value, without the large "wind up" and "power stroke" seen when a body frame is fixed to the middle link.

For real-world snake robots, we benefit primarily from the third trait. By applying the virtual chassis to the wide range of gaits we have developed, we have found that the orientation of the robot viewed in the virtual chassis corresponds closely to the way the robot interacts with the environment. Figure 3 shows the motion of the snake robot executing the sidewinding gait in a frame fixed to the head module and a frame aligned with the virtual chassis.

4. Computational Accuracy

Working in minimum perturbation coordinates, or a practical implementation of them, also offers benefits with regard to the accuracy of computations that rely on linearizing of the system dynamics. Almost all interesting locomoting systems are non-linear, but the degree of non-linearity depends heavily on the system's coordinate representation. Because of this, the same traits from the previous section that provide representational benefits also provide computational benefits.

In simple planar systems, using minimum perturbation coordinates allows us to easily identify effective motions by identifying the net displacement produced by different gaits with area integrals of the system dynamics over regions of the shape space.[1] These area integrals require linearization of the dynamics, and so were previously only applicable to small-scale mo-

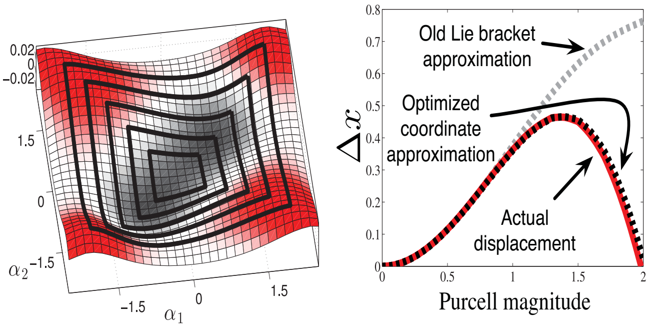

Fig. 5. Using the minimum perturbation coordinates greatly improves the accuracy of approximating the motion of a box stroke using the body velocity integral. Left, strokes of increasing magnitude in the shape space of the swimmer. Right, the displacement of the stroke shown in red, along with the approximation using a fixed frame (gray dashed) and minimum perturbation frame (black dashed).

tions, limiting their use in identifying maximally effective motions. Figure 4 shows the improved accuracy gained by using the minimum perturbation coordinates to suppress the nonlinearities of the three-link swimmer executing a box stroke. When using a minimum perturbation coordinate body frame, the body velocity integral remains an accurate approximation of the swimmer's true displacement, even for large magnitude strokes.

For snake robots, the virtual chassis allows us to fuse the redundant proprioceptive data distributed throughout the robot into a state estimate for pose using an extended Kalman filter (EKF).[8] One of the challenges of this technique is determining a process model for how the robot's pose changes over time. Models that are highly non-linear are problematic for an EKF because the linearization that the filter performs at each timestep becomes a poor approximation of the true system.[9] Working in the virtual chassis body frame not only improves our linearization, it allows a much simpler process model to be used in the first place. Because a true model of how the robot moves in the world is unknown, a constant velocity model is used as the prediction of body frame angular velocity between filter updates.

Normally, this model would be a poor predictor of the robot's motion, especially when the robot stops, starts, or changes its motions. Figure 4 shows the body frame angular velocities as the robot begins at rest, starts rolling rapidly, and then stops. This example shows how the dramatically smoother motion of the robot in the virtual chassis better matches the model's constant-velocity assumption, enabling the EKF to accurately esti-

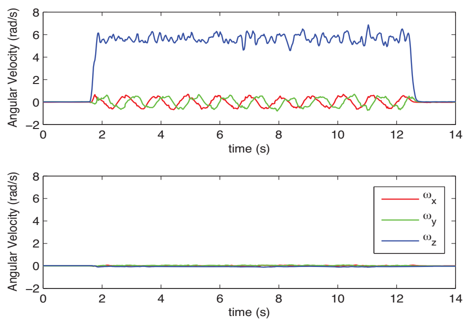

Fig. 6. A comparison of estimated angular velocities for the rolling gait, observed in a body frame fixed to the middle link (upper plot) and a body frame aligned with the virtual chassis (lower plot). The red, green and blue lines are respectively the x, y and z body frame angular velocities.

mate the robot's pose even during highly dynamic motions. This is because the virtual chassis separates the robot's internal shape changes (which are highly dynamic, but accurately measured with the robot's joint encoders) from it's external motions (which are less dynamic, and approximated with the constant velocity process model).

When using the virtual chassis the state estimator is also more robust to sensor errors, and far less sensitive to the filter's tuning parameters. Furthermore, the generality of the constant velocity model has allowed it to be successfully applied to a range of different gaits, including slithering, sidewinding, pipe crawling, and pole climbing.

5. Discussion

We have demonstrated the importance of coordinate choice in two domains of our lab's research, and shown that in each area the same underlying philosophies are at work. In particular, we have shown that when studying a locomoting system there are benefits from using a body frame that captures the system's overall motion, either by taking into account the system's dynamics or by simply averaging the system's kinematic configuration.

Using the virtual chassis lets us begin to treat articulated mobile robots more like wheeled robots, where models of the robot's motion in the world are much simpler and intuitive. For our lab's modular snake robots, we are currently working to build generic motion models using only knowledge of the robot's internal shape changes.[10] This is accomplished by averaging the movement of modules on the bottom of the robot, as defined by the flattest principle direction of the robot's shape in the virtual chassis. As a tool for designing new motions for the robot, the virtual chassis allows us to easily observe the macroscopic motion of the robot, without having to iteratively test the motions on a real robot or in physical simulator that models ground contact.

We also believe the virtual chassis is applicable to many systems beyond snake robots and planar locomoting systems. In general, we feel that any system that lacks a fixed point that is representative the system's overall motion can benefit from using the virtual chassis.

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