Predicting the Size of Spring Network Swarm in Quadratic Potential Fields

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Abstract - Formation control of multiple autonomous agents or a swarm of robots have become popular in robotics. Formation control is to maintain specific connections among multiple autonomous robots while performing tasks such as traversing trajectories, exploring environments, or covering spaces. We assume that robots are virtually connected by Delaunay triangulation, which is a general form of acute angle connection. This virtual connection forms a spring network, and we find tight upper bounds of the swarm size when we apply quadratic potential fields to the group of robots for the position and formation control. We present a method of Delaunay triangulation using circle bounds. By assuming hexagonal shape of each cell of swarm connection, we present a method of predicting a tight size of an ellipse that encircles the swarm.

Keywords - Swarm, Delaunay triangulation, Spring network, Quadratic potential

1. Introduction

Formation control of multiple autonomous agents (or a swarm of robots) have received a lot of attention due to recent advances in wireless communication, various micro/mini sensor technologies, and computing power. Formation control problem is defined as designing a control algorithm that maintains specific connections among multiple autonomous robots while performing tasks such as traversing trajectories, exploring environments, or covering spaces.

The main focus of this paper is to find a tight upper bound of the swarm size along with its shape for a specific connection between agents. Our approach build upon previous work in distributed manipulation with quadratic potential force fields, in which linear force fields are used for manipulating objects. We model a swarm in such a force field as a deformable object using spring network. Although employing a spring network model and a quadratic potential field is indeed very similar to considering virtual leaders and followers [1], and it is prevalent to adopt potential fields for formation control of swarm, we want to point out that only a few articles address the problem of predicting the size of the swarm.

Our method makes use of Delaunay triangulation based on local information for establishing sensor network. We predict the upper bound of the swarm size in a quadratic force field by computing density of the network at equilibrium state. Once we know the size of the swarm, in future work, we use ellipse-GVG to perform path planning for the swarm.

In the following section, we briefly review related work. In Section 3, we introduce a method of finding Delaunay triangulation. In Section 4, we compute the size of swarmming by analyzing the pressure distribution throughout the spring network. We discuss related future work in Section 5.

2. Related Work

For formation control of multiple agents, there are four popular approaches: 1) bio-minic behavior-based control, 2) leader-follower systems, 3) physics imitation, and 4) switched systems. Many of work uses combinations of these approaches.

Bio-mimic behavior-based control usually relies on simple local behaviors which adapts distributed controllers and generate large-scale properties of the system. Many researchers for example, Balch [2], Mataric [3] verify great adaptability and robustness of behavior based approaches to time-varying environment with relatively simple algorithms, as well as corresponding low computation cost during real-time operations.

Leader-follower approaches maintain either the explicit shape of the formation [4] or topological/statistical shape of the connection [5], [6]. Lynch [5], Belta [6]'s statistical formation controls utilize a virtual leader. Elkaim *et. al.* [7] use one leader and many followers to form a one dimensional contour with a leader at the center of the contour. If the number of follower robot increases, the size of the swarm become unnecessarily bigger than the case of two dimensional formation. Also, because followers only connect to two other neighbors, it may have trouble to pass through very narrow corridor where only one robot can traverse at a time. Formation control based on potential field method is nicely applied by Leonard [1] which consider leaders and followers in limited ranges. However, limiting interactions to local neighbors may cause disconnected clusters and does not guarantee a reconnection of the system.

Physics imitation method is prevalent such as adapting virtual potential fields [8], [9], virtual spring network [10]. Most of work with switched systems use graph representations to show the stability of networks using nearby-neighbor rules. Acute angle switching algorithm allows to connect all agents in view [11]. This method has no prediction on formation or the size of the swarm.

3. Delaunay Connection

Let $P = \{\vec{p}_1, \vec{p}_2, \cdots, \vec{p}_m\}$ be a set of robot locations (sites) in the two dimensional plane. A *Voronoi* region is defined by all the points at least as close to $\vec{p}_i$ as to any other site, *i.e.* ,

$$V(\vec{p}_i) = \{\vec{x} : \|\vec{p}_i - \vec{x}\| \leq \|\vec{p}_j - \vec{x}\| \forall j \neq i\} \tag{1}$$

Voronoi diagram $\mathcal{V}(P)$ is dual to Delaunay triangulation $\mathcal{D}(P)$.

Theorem *1:* For sites a and b, $\overline{ab} \in \mathcal{D}(P)$ if and only if there is an empty circle through a and b. The closed disk bounded by the circle contains no sites of P other than a and b [12].

Corollary *1:* For sites a and b, if the smallest circle that passes through a and b is empty, then $\overline{ab} \in \mathcal{D}(P)$.

We call such edges "strong" Delaunay connections. If a Delaunay edge is not strong Delaunay connection, it is called "weak" Delaunay connection. This is the case when the intersection of a Voronoi edge and the corresponding Delaunay edge is empty.

Corollary *2:* Strong Delaunay edge connection is equivalent to the acute angle connection (or meshes) which is proposed by Shucker in [11].

The acute-angle connection consists of edges between vertices A and B such that for all other verices C, the interior angle $\angle ACB$ is acute. Obviously, this test is equivalent to Corollary 1.

Algorithm *1:* Given a set of robots around p_i, $P_i = \{x : \|p_i - x\| \leq R_{max}\}$ where R_{max} is the maximum sensor range for a robot site p_i, find Delaunay connection, $\mathcal{D}(p_i)$:

1) $\forall p_j (j \neq i) \in P_i$, if $O(p_i, p_j)$ is empty, $\overline{p_i p_j} \in \mathcal{D}(p_i)$
2) $\forall p_k (k \neq i, j) \in P_i$ and $\forall \overline{p_i p_j} \in \mathcal{D}(p_i)$, if $O(p_i, p_j, p_k)$ is empty, $\overline{p_i p_k} \in \mathcal{D}(p_i)$

where $O(a, b)$ is the smallest circle passing through two sites a and b, whereas $O(a, b, c)$ is the circle defined by three sites a, b, and c.

Notice that the step 1) generates "strong" Delaunay connection and the step 2) completes the Delaunay connection for p_i by adding weak Delaunay connections (if there exist) in the strong Delaunay connection. The complexity of the algorithm increases with the number robots within a local communication region, but is NOT dependent on the number of robots out of communication range.

4. Swarm Formation in Quadratic Potential Fields

We consider the case that each agent can detect at least one other robot in the group, *i.e.* , a connected network. In such a case, each robot has information of the relative location of its neighbor, thus it is reasonable to assume that each robot has information about its location relative to the mass center. [cite a paper finds a center of the mass based on local information]

In this section, we briefly introduce quadratic potential force fields and predict the size of a spring network in such force fields.

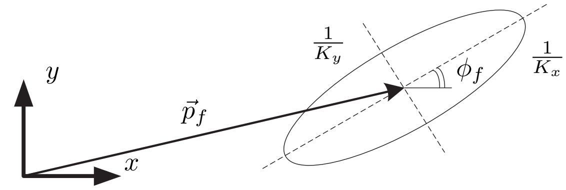


Fig. 1. Coordinate transformation to the field, specifying the shape (K_x and K_y), position $\vec{p}_f$ and orientation ϕ_f of the (in this example) elliptical field.

4.1. Quadratic Potential Fields

We briefly review quadratic potential force fields. The general form of a quadratic potential is:

$$U(\vec{p}) = \vec{p}^T \underbrace{\begin{bmatrix} c_{xx} & c_{xy} \\ c_{yx} & c_{yy} \end{bmatrix}}_{\hat{c}} \vec{p} + \begin{bmatrix} c_x & c_y \end{bmatrix} \vec{p} + c \tag{2}$$

where $\vec{p} = \begin{bmatrix} x & y \end{bmatrix}^T$ is a point in global coordinates. The force acting on a point robot at (x, y) is the negative gradient of Equation 2 which with an eigenvalue decomposition becomes:

$$\vec{F}(\vec{p}) = -\nabla u(x, y) = -R \underbrace{\begin{bmatrix} K_x & 0 \\ 0 & K_y \end{bmatrix}}_{K} R^{-1} \vec{p} - \vec{f}_o \tag{3}$$

where $R = \begin{bmatrix} \cos\phi_f & -\sin\phi_f \\ \sin\phi_f & \cos\phi_f \end{bmatrix}$ is a rotation matrix (with angle ϕ_f determining the orientation of the field), K_x and K_y are the eigenvalues and represent the strengths of the fields in the two directions and $\vec{f}_o$ is the force produced by the field at the origin and it encodes the location $\vec{p}_f$ of the field defined uniquely as the point where $\vec{f}(\vec{p}_f) = 0$. Thus any quadratic field can be represented by exactly five parameters:

$$Q = \{K_x, K_y, \phi_f, \vec{p}_f\} \tag{4}$$

For the sake of simple discussion, we assume $\phi_f = 0$, $f_o = 0$, and $\vec{p}_f = \vec{0}$. Also, in this paper, because we are interested in predicting the size a swarm in a quadratic field, we assume $K_x K_y > 0$, *i.e.* , elliptic fields. Considering hyperbolic fields ($K_x K_y < 0$) for swarm formation is a future topic.

4.2. Spring network in elliptic potential force fields

We use a spring network model to study a swarm formation problem. We are interested in predicting an upper bound of spring network size in elliptic potential force fields.

When two robots $\vec{p}_i, \vec{p}_j$ are within a communication range R_{max}, they are connected each other by a virtual spring with stiffness $k_{ij} = k_{ji}$ and norminal length d. The equilibrium pose of the swarm is obtained when the gradient of the total energy is zero. Total energy $V_{total} = V_{spr} + V_{pot}$, *i.e.* ,

$$V_{spr} = \sum_i^N \sum_j^N \frac{k_{ij}}{2} \left(\|\vec{p}_i - \vec{p}_j\| - d\right)^2 \tag{5}$$

$$V_{pot} = \sum_i^N \frac{1}{2} \vec{p}_i^T K \vec{p}_i \tag{6}$$

where $k_{ii} = 0$.

Note that because we do not know the spring length at equilibrium, minimizing the total potential energy is not our interest. Instead, we find the case that

$$\frac{\partial V_{total}}{\partial \vec{p}_i} = \vec{0} \tag{7}$$

Notice that

$$\frac{\partial V_{spr}}{\partial \vec{p}_i} = \sum_j^N k_{ij}(\|\vec{p}_i - \vec{p}_j\| - d)\hat{e}_{ij} \tag{8}$$

where $\hat{e}_{ij} = \frac{\vec{p}_i - \vec{p}_j}{\|\vec{p}_i - \vec{p}_j\|}$. Eq. 7 becomes

$$K\vec{p}_i + \gamma_i \vec{p}_i - \sum_j^N k_{ij}(\vec{p}_j - d\hat{e}_{ij}) = \vec{0}, \tag{9}$$

where $\gamma_i = \sum_j^N k_{ij}$.

It is easy to see that the first moments of area (mean position) of the swarm are zero. As $\hat{e}_{ij} + \hat{e}_{ji} = \vec{0}$, $\hat{e}_{ii} = \vec{0}$, and $k_{ij} = k_{ji}$, we get

$$\begin{aligned} N\vec{\mu} &= \sum_i^N \vec{p}_i = K^{-1} \sum_i^N \left(\sum_j^N k_{ij}(\vec{p}_j - d\hat{e}_{ij}) - \gamma_i \vec{p}_i \right) \\ &= K^{-1} \left\{ \sum_i^N \sum_j^N (k_{ij}\vec{p}_j - k_{ij}\vec{p}_i) - d \sum_i^N \sum_j^N \hat{e}_{ij} k_{ij} \right\} = \vec{0} \end{aligned}$$

In many articles, it is shown that the equilibrium pose of swarm in a potential field is at the statistical mean of the swarm. [need some references]

4.3. The size of swarm

To predict the size of the swarm (spring network model) in elliptic potential force fields, we assume the following.

1) Robot connection is established based on a Delaunay diagram whose edges are springs.
2) Robot connection is established when $d < R_{max}$ where d is the distance between two robots and R_{max} is the sensor range.
3) The aspect ratio of the swarm is the same as that of the elliptic force field. (or equivalently, the shape of swarm is the same as that of the elliptic field.)
4) Ignore discreteness.
5) Homogeneous robots.

The first assumption provides us all the nice properties of planar graph. The second assumption implies that the virtual spring connection is established only when two robots are within their sensor range. The third assumption is related to the shape of the outermost rim. Suppose that some of the outermost robots are located outside of the equipotential line as in Fig. 2. Because the equipotential line is an ellipse, there are equal number of inward dents (region A in Fig. 2) and outward bumps (region B in Fig. 2). When the system is in equilibrium, the moment about point C_2 must be zero, thus the outermost line of the robots must be aligned with the equipotential line.

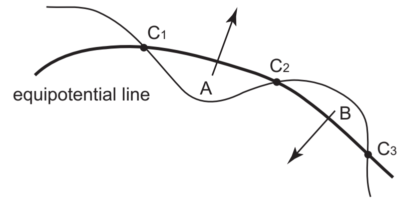


Fig. 2. The shape of the swarm in the elliptic potential force field

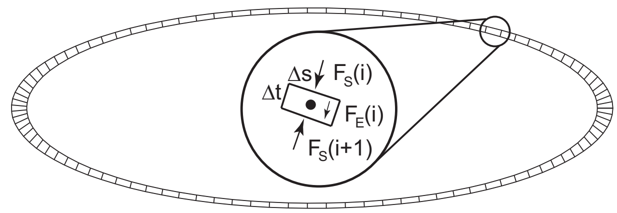


Fig. 3. Constant pressure on the equi-potential contour

This argument is valid for infinite number of robots, thus we consider the elliptic shape of the swarm as an assumption.

Without loss of generality, suppose that the center of the elliptic potential force field at the origin and the field is aligned with the axis. Thus, the elliptic force at $\vec{p} = (x, y)$ is that $\vec{F}(x, y) = (F_x, F_y)$, *i.e.*,

$$F_x = -K_x x \tag{10}$$

$$F_y = -K_y y \tag{11}$$

where (x, y) is on a equi-potential ellipse α.

In order to predict the size of the swarm, we first find the normal components of the pressure forces exerting on a robot at equilibrium along the elliptic perimeter. We consider an infinitesimal element along the parametrized ellipse perimeter whose thickness Δt and the width are functions of the difference of levels, $\Delta\alpha = \alpha_i - \alpha_{i+1}$. The thickness Δt is normal to the ellipse, *i.e.*, Δt is taken along the direction of $\vec{F}$.

$$\Delta t = \frac{\sqrt{K_x K_y \xi}(\xi - \sqrt{\xi^2 - \eta\Delta\alpha})}{\eta} \cong \frac{\sqrt{K_x K_y}}{2\sqrt{\xi}}\Delta\alpha \tag{12}$$

$$\Delta s = \sqrt{\xi}\Delta\theta \tag{13}$$

$$F_E(i) = \sqrt{K_x K_y \eta} \tag{14}$$

where $\xi = \alpha_i(K_x \cos^2\theta + K_y \sin^2\theta), \eta = \alpha_i(K_x^2 \cos^2\theta + K_y^2 \sin^2\theta)$, $F_E(i)$ is a body force at equi-potential α_i (quadratic potential force per unit robot).

$$F_E(i)\Delta t = \frac{\xi(\xi - \sqrt{\xi^2 - \eta\Delta\alpha})}{\eta} K_x K_y \cong \frac{K_x K_y}{2}\Delta\alpha \tag{15}$$

Notice that the density along the equi-potential ellipse is constant (because the area of the infinitesimal element is constant) and so is $F_E(i)\Delta t$ which increases as it goes inside. (Remember that $\alpha_{i+1} = \alpha_i - \Delta\alpha$.) Furthermore, because the exerting force is normal to the ellipse, *i.e*, pure pressure on the ellipse, we have the pressure balance between the spring pressure difference and the elliptic potential force pressure,

$$\Delta p_s = p_s(i+1) - p_s(i) = \rho(\alpha_i) F_E(i) \Delta t \tag{16}$$

where $\rho(\alpha_i) = \frac{r}{l_s(\alpha_i)^2}$ is the robot density.

From the pressure balance on the elliptical strip,

$$p_E = \rho(\alpha_i) F_E \Delta t = \frac{r K_x K_y \Delta\alpha}{2 l_s(\alpha_i)^2} \tag{17}$$

$$\Delta p_s = \frac{k(l_0 - l_s(\alpha_i))}{w l_s(\alpha_i)} - \frac{k(l_0 - l_s(\alpha_{i-1}))}{w l_s(\alpha_{i-1})} = \tag{18}$$

$$k l_0 \left(\frac{1}{w l_s(\alpha_i)} - \frac{1}{l_s(w \alpha_{i-1})} \right) \tag{19}$$

where w is the length parameter for the spring pressure.

we get the equilibrium spring length :

$$l_s(\alpha_i) = \frac{l_s(\alpha_{i-1})}{2} + \sqrt{\left(\frac{l_s(\alpha_{i-1})}{2}\right)^2 - Y l_s(\alpha_{i-1})} \tag{20}$$

where $Y = \frac{rwK_xK_y\Delta a}{2kl_0}$, $l_s(\alpha_0) = l_0$. Thus, we know the density distribution of the swarm if we know the shape of the spring connection.

The total mass (number of robots) can be computed from the density function,

$$m(\alpha) = \int\int \rho(\alpha_i) \Delta s \Delta t = \int\int \frac{r}{l_s(\alpha_i)^2} \Delta s \Delta t \tag{21}$$

We can compute the size of the swarm iteratively. For an initial guess on α, we compute the total mass, check the difference between the estimated total mass and the real number of robots, and iterate α to recompute the total mass.

In the iterative process, the integral is taken from the outer bound to inward because the robots in the outermost part are connected by normal length of the spring which is known priorly. A naive initial value for the potential level α would be the smallest ellipse that encircles the straight line of the length Nl_s. We present an iterative method of predicting the size of a spring network in an elliptic potential force field with a better initial condition.

We assume the hexagon connection of the spring network because the spring network will form such a connection when infinitesimal quadratic potential field is applied to the system. When relatively strong potential force field is applied, the inner part of the spring connection experiences more pressure than the outer part. Also, the network connection is allowed to change as the robot position changes. Thus, the assumption of the hexagonal connection over-estimates the size of the swarm. In case of the hexagon connection, the density parameter (r) and the length parameter (w) for the spring pressure are assumed to be $\frac{2}{\sqrt{3}}$ and $\frac{1}{\sqrt{3}}$ respectively.

Proposition *1:* Robot swarm in the elliptic potential field(without loss of generality, we assume that $K_x = \rho K_y > K_y$) is bounded by the ellipse of $E_y = \frac{\sqrt{3}Nd_{eq}^2}{2\pi\sqrt{\rho}}, E_x = \rho E_y$ where N is the number faces in the Delaunay diagram, and ρ is the aspect radio of the imposed elliptic potential field, and $d_{eq} = \frac{\sqrt{3}kd}{\sqrt{3}k+\sqrt{K_xK_y}}$ is the length of hexagon edge at equilibrium.

When very small amount of unit radial force field is imposed to the swarm such that each robot is connected to at least one other robot and the connecting spring is compressed ϵ amount, the swarm forms a hexagon array. Due to the assumptions, the robot swarm forms a Delaunay triangulation (or equivalently Voronoi graph). The upper limit of the total area of the swarm at the equilibrium is $\frac{\sqrt{3}}{2}Nd_{eq}^2$ where d_{eq} is the length of the spring network at equilibrium. The assumption on the shape of the swarm makes the area of an ellipse become $\pi E_x E_y = \frac{\pi}{\alpha}\sqrt{K_xK_y}$ where $E_x(E_y)$ is the length of semi-major(minor) axis of the ellipse that enclosing the swarm and α is its potential constant. The spring length at the equilibrium d_{eq} is approximately found as follows.

Because of the hexagonal shape assumption, the pressure due to the spring force (spring force per unit length) is that $p_h = \frac{\sqrt{3}(d-d_{eq})k}{d_{eq}}$. At the equilibrium, this internal pressure is equivalent to the external pressure due to the elliptic force field, $p_e = \frac{F_p}{L_p}$ where F_p is the total force along the equi-potential perimeter and L_p is the perimeter of the equi-potential line. We use Euler approximation for the perimeter of the ellipse, *i.e.* , $L_p = 2\pi\sqrt{\frac{E_x^2+E_y^2}{2}}$ and $F_p = 2\pi\sqrt{\frac{\alpha K_xK_y(K_x+K_y)}{2}}$. Thus, $p_e = \sqrt{K_xK_y}$ and $d_{eq} = \frac{\sqrt{3}kd}{\sqrt{3}k+\sqrt{K_xK_y}}$.

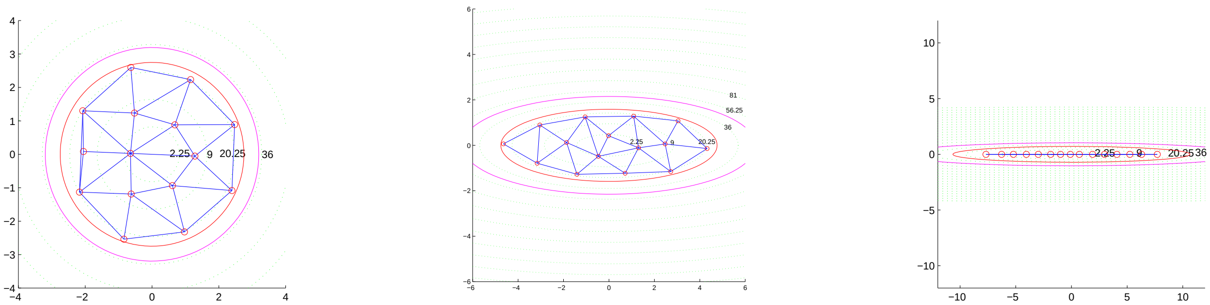
Note that when two robots in the elliptic force field where $K_x > K_y$, the equilibrium spring length becomes $\frac{kd}{k+K_y}$. This ellipse is an upper bound for the size of the swarm in equilibrium because the size of hexagons are not uniform in real situation. The inner pressure is bigger than the outer pressure, thus the inner equilibrium spring length is smaller than the outer one.

Proposition *2:* Initially set α from Proposition 1 and compute the total mass of the spring network using Equations 20 and 21 to check it with the real number of robots. Reduce α and repeat computing the total mass until it is the same as the real number of robots.

Notice that the size of the bounding ellipse can be pre-computed because it is a function of the spring constant, the normal length of the spring, the strength of the elliptic force field, and the number robots. As the predicted size is not dependent on which type of optimization method is used, such a topic is not discussed in this paper.

Example *1:* As the aspect ratio of the elliptic potential force field changes from 1 to smaller number, the shape of the swarm changes from a circle to thinner ellipse. When $K_x << K_y$, as the y-directional squeezing force dominates, the swarm forms a string aligned on the x-axis. Fig. 4 show three cases of the aspect ration.

Note that the predicted size of a swarm is an upper bound, not necessarily a tight bound. When $K_x << K_y$, the formation of the swarm becomes a line, in which case we can compute the length of the connected line network using symmetry. However, our estimation assumes the shape of each cell of



(a) $K_x = K_y = 0.2$ (b) $K_x = 0.1, K_y = 0.9$ (c) $K_x = 0.02, K_y = 4.5$

Fig. 4. Examples of the size of swarm. The robot locations are marked by circles. The number of robots, $N = 15$, spring constant, $k_s = 1.1$, normal length of the spring, $l_0 = 2$. Dotted ellipses are equi-potential lines, the thin line is the initial condition and the thick line is the bound we found.

the network connection to be hexagon, thus we overestimate the size.

5. Future work and discussion

In this paper, we present a method of predicting the size of swarm which is modeled with spring network in quadratic potential fields. Knowing the tight upper bound of the swarm size is important especially for path planning of the robot group in a bounded environment. In our problem it is assumed that all the robot has a perfect knowledge of the group, knows exact location of other robots at every moment. Relaxing this condition is a future topic, where only robots on the boundary of the group can detect the distance to the environment. The measurements of these robots on the outer-most region can be shared with the information of neighboring robots in the group. Theoretically, if the sensor network of the swarm is fully connected, then all the robots will eventually have the full information of the environment after the information is fully propagated. In this case, however, the convergence rate of the information propagation could be slower than the agent's motion, thus the delayed formation could cause instability of the formation. Thus, it is better not to rely on the delayed full information of the network. Instead, each agents could use an estimator to "guess" a proper full information using neighboring robot's information.

Once we know the size of the group and its location in the closed environment, we can adopt an ellipse-GVG to plan a path. Path planning for a planar ellipse body is well studied topic. In addition, we have a degree of freedom to reform the group, in other words, the group of robot can be separated into several smaller groups or two groups can be merged to form a bigger group. In each case, a path plan can be done for a leader group and the other smaller groups can just follow the leader. Realizing the group merging and breaking group is a future topic along with path planning for a swarm.

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